Chapter 8
Simulation Methods: Outline

Thierry Kamionka

Abstract This chapter is devoted to presenting estimation methods based on simulations in econometrics. The techniques used are based on concepts that were discussed very early on, but the first significant advances emerged as a result of the availability of computing resources. Developments specifically for econometrics appeared mainly with simulated likelihood and simulated moments. Other methods then emerged at the same time as the first ones were being improved.

8.1 Introduction

- Many examples in econometrics of models that incorporate at least an integral that is complicated to evaluate for a given value of the parameter.
- There may be omitted heterogeneity that requires integration of the conditional density with respect to the density of the distribution of this heterogeneity.
- Simulation-based estimation methods make it easier to estimate the parameters of a model in cases where the objective functions of conventional estimators are complex to optimize.
- This is the case when the individual must choose between different states in order to maximize his expected discounted utility within the framework of structural modeling.
- However, simulation methods can be used in other fields to evaluate quantities such as probabilities. The first example of estimation by

Thierry Kamionka ✉
CNRS, CREST and Institut Polytechnique de Paris, France, e-mail: kamionka@ensae.fr

simulation was proposed by Pierre-Simon Laplace (1812) to estimate the number π and is based on an experiment initially proposed by Georges-Louis Leclerc de Buffon (1733). This experiment is called the Buffon's needle.

8.2 Monte Carlo Method and Bootstrap Method

Topics :

- Monte Carlo Method : Metropolis and Ulam (1949), Metropolis (1987)
- Bootstrap method: Efron (1982), Efron and Tibshirani (1993)
- Importance sampling: Kloek and van Dijk (1978)

Lessons :

- A Monte Carlo method allows to evaluate multi-dimensional integrals. It is based on a numerical technique which uses random draws.
- Importance sampling is a Monte Carlo method that allows draws to be made from a distribution for which they are easy to obtain.
- Bootstrap allows to estimate the sampling distribution of a statistic from the observed sample.
- Monte Carlo methods should now be considered as elements of the estimation methods presented in the other sections. Is bootstrap widely used.

8.3 Simulated Maximum likelihood Estimator

Topics :

- The use of simulated frequencies to evaluate choice probabilities : Lerman and Manski (1981)
- GHK algorithm : Geweke (1989), Hajivassiliou (), Keane (1990, 1994).
- Combining continuous and discrete variables : Chang (2011), Brunet, Kamionka and Lacroix (2024)
- Estimating the parameters of a model defined in continuous time from discrete time observation : Kamionka (1998)

- Smoothing likelihood function using simulated histories : Keane and Sauer (2009)

Lessons :

- Easy to implement.
- Good properties in practice even with a fixed number of draws.
- Allow to control the approximation made for the evaluation of a given integral
- Adapted to simulation of high-order multiple integrals but also simple integrals.
- A lot of application in literature on applied econometrics.
- Simulated maximum likelihood estimation is commonly used (e.g., consumption choices). It is noteworthy that these properties are considered good in practice even for a fixed number of draws, provided that they are specific to individuals.

8.4 Method of Simulated Moments

Topics :

- Generalized method of moments: Hansen (1982)
- Method of simulated moments : McFadden (1989, 1996)
- A structural approach consisting to model a vector of ergodic stationary random variables : Michner 1984, Duffie and Singleton (1993)

Lessons :

- Allows to conduce estimation even if there no closed form for the moments.
- Suitable for a semi-parametric modeling.
- Many applications in econometrics and, in particular, in the fields of financial econometrics and consumer choice.
- Possibility to use the approach under the assumption we consider an ergodic and stationary process.

8.5 Indirect Inference

Topics :

- Calibration based on score: Gallant and Tauchen (1996)
- Calibration based on comparison of estimators: Gouriéroux, Monfort, Renault (1993)
- Comparison of these estimators : Gouriéroux and Monfort (1995)
- Solutions to remove the original difficulty: 1) Keane and Smith (2003) propose a change of variable technique. 2) Bruins et al. (2018), Frazier et al. (2019) propose a smoothing technique.
- Calibration based on the comparison of the likelihood functions of auxiliary model : Keane and Smith (2003).‘

Lessons :

- A way to simplify the estimation using a simple auxiliary model.
- Applications in the field of micro econometrics, financial econometrics.
- A difficulty arose very quickly: when one at least of the dependent variable is discrete. In this case the objective function is not continuous.
- However, several solutions are now available for model with discrete dependent variable.
- This estimation method is relatively old but remains useful because additions have been made to increase its scope.

8.6 Simulated EM algorithms and Estimation

Topics :

- EM algorithm: Dempster, Laird, Rubin (1977)
- Sim EM algorithm: Ruud (1991), Train (2008)
- Stochastic EM algorithm : Celeux and Diebolt (1985)
- Expectation-Conditional Maximization (ECM: Meng and Rubin (1993) and Arcidiacono and Bailey Jones (2003)

Lessons :

- The EM algorithm allows the ML estimator to be obtained.
- Easy to implement.
- In the case of SimEM and StEM algorithms, the asymptotic properties are known.
- Applications in micro-econometrics.

- Allow to estimate ”large models” with unobserved heterogeneity : Arcidiacono and Jones (2003), which exploits conditional separability (ESM algorithm).
- These estimation methods are more recent and have benefited from the fact that the asymptotic properties are now established.

8.7 Dynamic Discrete Choice

Topics :

- Illustration using a model of schooling : Eisenhauer, Heckman and Mosso (2015)
- Backward induction : Bellman (1957)
- Estimation : MLE vs. MSM.
- Classification error : Keane and Sauer (2009)

Lessons :

- Moments are replaced by empirical average obtained generating trajectories of decisions.
- Adapted to structural modeling.
- Importance sampling allows to obtain a continuous objective function.
- The application of simulation methods in a dynamic setting often relies on the MSM estimator. The use of indirect inference methods is an interesting alternative.

8.8 Nonparametric Simulated Maximum Likelihood

Topics :

- Static parametric models : Fermanian and Salanié (2004)
- Dynamic models : Kristensen and Shin (2012)

Lessons :

- Adapted for contexts such that the conditional density of the dependent variable does not have a closed form.
- Dynamic models can include non stationary and time-inhomogeneous processes.

- The estimator behaves in such a way that there is convergence and asymptotic normality.
- Can be used in a context of the individual choices are described by optimization behaviors (Tincani, 2021).
- This is a recent method.

8.9 Looking ahead:

Topics :

- Challenge: Structural models that are more realistic because they are based on less restrictive assumptions (e.g., non-stationarity) but are complex to estimate using data.
- Models based on many equations with multiple error terms and numerous individual effects.
- Models based on conditional quantiles.

Lessons :

- The toolbox provided by simulation estimation methods can be used to address the challenges posed by more complex models.

8.10 Conclusion

- Simulation based estimation techniques provide ways to estimate model parameters more easily.
- This may be the case when the likelihood function involves multiple integrals, or when the model has many parameters, or because the model is based on assumptions about individual behavior such that we do not have the expression for the conditional density of the endogenous variable.

References